Brief lecture notes on the discipline:
Introduction to the theory of supersymmetry

№1 Continuous integrals and point particles

The combination of two fundamental theories of modern physics, quantum field theory and the general relativity, within the framework of a single theoretical approach, is one of the most important unresolved problems. It is noteworthy that these two theories, together taken, embody the full amount of human knowledge about the most fundamental forces of nature. Quantum field theory, for example, has achieved extraordinary success in explaining the physics of the microworld up to distances not exceeding 10^{-15} cm. The general relativity (GR), on the other hand, has no equal in explaining the large-scale behavior of the cosmos, giving a beautiful and A fascinating explanation of the origin of the universe itself. The striking success of these two theories is that together they can explain the behavior of matter and energy in a staggering range of magnitudes of 40 orders of magnitude, from the subnuclear to the cosmological domain.

The great mystery of the last five decades, however, was the complete incompatibility of these two theories. It looks as if nature had two minds, each of which operates independently of the other in its field, acting in complete isolation from each other. Why is nature, at its deepest and most fundamental level, to require two completely different approaches with two sets of mathematical methods, two sets of postulates and two sets of physical principles?

Ideally, we would like to have a unified field theory, combining these two fundamental theories:

Quantum Field Theory and General Relativity - Unified Field Theory. However, the history of attempts to unite these two theories in the past decades has been sad. They invariably shattered due to the appearance of infinities (divergences) or violated some revered physical principles, such as the principle of causality. The powerful methods of renormalization theory, developed in quantum field theory over the past decades, have not been able to eliminate the divergence of the quantum theory of gravity. It is clear that the most important piece of the puzzle has not yet been found.

Quantum field theory and general relativity - unified field theory. However, the history of attempts to combine these two theories in the past decade was sad. They invariably shatters due to the appearance of infinities (divergences) or violated some revered physical principles such as the principle of causality. Powerful methods of renormalization theory developed in quantum field theory in the past decade have not been able to eliminate the divergence of the quantum theory of gravity. It is clear that the most important piece of the puzzle has not yet been found.

Although quantum field theory and general relativity seem completely incompatible, the last two decades of intensive theoretical research have made it increasingly clear that the secret of this mystery most likely lies in the power of gauge symmetry.

Although quantum field theory and general relativity seem completely incompatible, the last two decades of intense theoretical research have made increasingly clear that the secret of this mystery probably lies in the power of the gauge symmetry. One of the most remarkable features of nature - the fact that its basic laws have majestic unity and symmetry when they are expressed in the language of group theory. Combining with the help of the gauge symmetry is undoubtedly one of the most instructive lessons taught physics. In particular, the use of local symmetries in Yang-Mills theory led to great success in the fight against divergences in quantum field theory and in bringing the laws of physics of elementary particles in an elegant and comprehensive approach. Nature, it seems, does not just include symmetry in physical laws for aesthetic reasons. Nature requires symmetry.

The problem, however, was that even the powerful symmetries of the Yang-Mills gauge theory and the general covariance of Einstein's equations turned out to be insufficient to obtain the quantum theory of gravity free of divergences.

№2 Secondary quantization. Harmonic oscillators

So far we have examined only the first quantized approach to quantum particles. Only the quantized vectors of position and momentum:

The first quantization: \ _r$_s$ x_j] = - (1-

The limitations of this approach, however, will soon become apparent as soon as we introduce interaction. Suppose we want to describe the point particles that can collide with each other and break down Xia, not only move in the external potential. Now we have to modify the generating functional to include summation over Feynman diagrams:

Z = X $ Dxe ~$^{an\,\prime\prime Ha}$. (1.7.2)
topology

(Note that we did Wick rotation in the integral over t to exhibitor converge. Since the exponent becomes substance--governmental, from the context it will be clear that in this book refers to the theory with this turn. We will not discuss the delicate issue of convergence path integral.)

In other words, we have to manually summed over all the topologies of the particles, if the particles can be split and converted-inrush. Each topology is the time evolution traek thorium-all point particles in the course of interaction. The amplitude of the LH-chas-
adic scattering with momentum generated by the set of k$_{l9}$ *to*$_2$, ..., k_N.
It can now be written as

A ($k_{l9}k_{29}...{}_9k_N$*)* = I0 ^{I}jD.x; A$_F$p
topology

X exp | - fdi L (t) + i £ / c$_u$ 4 J. (1.7.3)

Note that we take the Fourier transform of the Green's function, so that the amplitude is a function of external impulses. This formula can be conveniently represented in the form

/
N X <e x p; £ ^>. (1.7.4)
topol
ogist uu i = 1

Thus, we assign a factor e1kh each foreign particle. It is derived from the corresponding term of the Fourier transform. The formula for the scattering amplitude derived by the path integral, important that almost no change is transferred to the string theory formalism.

№3 Nambu-Goto

String theory seems at first sight deviating from the conventional methods developed over the last 40 years for the theory of second-quantized fields. This is because string theory has historically been first discovered in the form of first quantization theory. That is why string theory sometimes looks like a random collection of arbitrary assumptions. Although the theory of second quantized field can be completely withdrawn from the expressions for the only action of the primary quantization theory require additional assumptions. In particular, the vertices, the choice of the interactions and the weight of the perturbation theory diagrams must be postulated by hand and then checked on the unitary.

Fortunately, the path integral formalism for the first quantized point particle was then extended to the string Gervais and Sakit, which allows us to write the dynamics of interacting strings with remarkable ease.

In the previous chapter, we introduced the most important mathematical concepts, on the basis of which it is possible to discuss the theory of the primary quantization-tion for point particles. Oddly enough, almost all of the basic features of Nambu-Goto have those or other analogues in this theory. Of course, in string theory there are quite new features, such as the existence of powerful symmetries of the world-sheet, but the basic methods of quantization can be directly transferred to the case of a point particle, discussed in the previous chapter.

We have seen that the usual formulation of the second quantized field theory can be rewritten in the form of first quantization. So, the traditional covariant Feynman propagator (1.6.19)

by using (1.3.28), (1.3.30), (1.3.37) in the form

$A_F(x_{lf} \quad x_2)$
$= \quad (\quad 2\ ^x rV$
$- r (n -$
$= \quad Dx \quad t^2)_>$

=] f Dxe- °/ 2) S ldi {i '-
di 2 m \ (2.1.1)

where we integrate over all possible trajectories of the particles, finding the point scheysya-hee (t), which begin at x, and terminate at a point x2.

Interaction, as we have seen, introduced into the theory manually postuliovaniem particular set of topologies in which the particle can blu * Dat- scattering amplitude, for example, have

$A\ (k_1, k_2, .., k_N) = IjDxe- \wedge + \quad \wedge '- Arr$
TOPOLOGY

N
= X <P e ' (2.1.2)
topology 1 *

where we integrate over all topologies, forming the famous Feynman diagrams for the theory F3 or F4.

It is important to note that the resulting Feynman diagram is a graph rather than a variety. At the point of interaction between local topology is not R ", so that it can not be manifold. There is no correlation between the internal lines and points of interaction. This means that we can enter at the point of interaction of relativistic point particle arbitrarily high back, if we use formalism first quantization. Therefore, the first quantized theory of a point particle has an infinite degree of arbitrariness, which corresponds to the different spins and masses, which we can put to the interaction point. Cr IU of ultraviolet divergence Feynman diagram each correspond to number of ways one can "Prischepa" this diagram styanuv in any point of the internal lines,

№4 From the path integral to the operators

We are used to calculate the K-point amplitude of functional methods. The only non-trivial part of the calculation was the integration of the two-dimensional conformal nonequivalent complex surfaces, which determines the amplitude measure of integration.

The same calculation can be performed using the formalism of harmonic oscillators, which, as we have emphasized, is a specific representation of the path integral for which the Hamiltonian is diagonal. For trees and the first loop method of harmonic oscillators is very simple, because the Hamiltonian is diagonal to the Fock space of harmonic oscillators. However, for higher loops it is not so: the method of harmonic oscillators becomes more time-consuming and impractical. Path integral therefore provides the only systematic way to study the amplitudes of higher loops with relative ease.

(Calculation of anomalies, however, it is easier to carry out in the formalism of harmonic oscillators, where n is an integer index theory cutoff parameter. In the formalism of continual integrals la necessary to use the method in terms of splitting and other ways of regularization, as we shall see in Chap. 5.)

We know that in the functional formalism propagator for free strings given by the formula

$$f_{abo} \quad xi> = \frac{}{L_0 - 1} \tag{2.6.1}$$

Similarly, we know that as a result of the insertion path integral in the full set of intermediate states ver. teksnaya function / -th tachyon takes the form

$$<x_a \backslash e^{ik} \wedge \backslash X_b> \tag{2.6.2}$$

Where

$$Xl = X_{[l}(O = 0, T = x_f). \tag{2.6.3}$$

due to the identity

$$\backslash X) \$ DX (X \backslash = \backslash$$

we can eliminate all the path integrals of LH-point function and make the transition from the formalism of a path integral formalism for harmonic oscillators. For example, start with the expression for N-point tachyon amplitude in the path integral formalism and substituting therein the appropriate shape-ly in terms of the harmonic oscillator:

$$An \quad \backslash DXd \backslash \; ie \sim^{s} \; \prod_{i=1}^{N}] -] hy \wedge i =$$

$$= fd\,[ie \sim^{s} \{\ldots \quad j\,DX\,j\,(Xj \backslash <> e^{ikj} + \; e \wedge * \wedge | \; \}f | \quad X\,j)\,f\,DXj\,(X\,j\,|\,e - * '*' '' \;{}^{T\prime}> (0)'' \; \bullet\}$$

$$= \{\ldots e^{ik_t X\{0)} \quad V^{fc\,''} \; \}$$

$$= \quad k_t \backslash e^{ik} 2^x M_e -^{f} >^{H} e^{ik_i X\{O)} \; \backslash 0, k_N> -$$

$$= \quad (0, k_1 \backslash V(k_2)\,DV(k_3) \ldots V(k_{N-1}) \backslash 0, k_N\}.$$

[illegible] odoibmroasztom insakmlyutachkizhte formalism, harmonic oscillator. The expression for the vertex function (2.6.2)

formally becomes infinite in the transition to a harmonic oscillator. For example, if we take the unsophisticated vacuum expectation value of the exponent, we

We obtain the outgoing amount. Normally ordered expression - that's what we need; its matrix elements are finite.

№5 Superstrings

Supersymmetry - the most elegant of all the symmetries; it unites bosons and fermions in one multiplet:

<Fermions - bosons *

Combining fields with different statistics, supersymmetry and supergroup also opened up a whole new field of mathematical research.

But, unfortunately, there is no experimentally established fact that would have testified in favor of this theory. For example, physicists have tried to find the supersymmetric multiplets for the electron-neutrino or throne, but it was not possible to detect scalar analogs of these particles. In fact, none of the currently known particle there is a supersymmetric partner. Some critics have called supersymmetry "solution for which you want to find the problem."

Although there is absolutely no empirical evidence justifying the introduction of the concept of supersymmetry, it is undeniable that this concept gives us a whole treasury is extremely desirable theoretical techniques that promise huge profits. Supersymmetry is something more than just an elegant way of combining elementary particles in the eye pleasing multiplets; it has some practical use in quantum field theory. Here's the list.

(1) Supersymmetry generates identities super-Ward-Takahashi, destroying many usually divergent Feynman diagrams. For example, loop Feynman diagrams with Bo-zones and fermions, circulating inside the loop differ by a factor - 1. Due SUSY boson loop can be reduced with the remaining Fermionic and divergence will be much milder. We see, therefore, that the Yang-Mills theory with supersymmetry renormalization have better properties than conventional gauge theories. Indeed, some "about nonrenormalizability theorem" can dock, to show in all orders of perturbation theory.

(2) Supersymmetry can solve the "hierarchy problem" that has become the curse of the type of grand unified theories (GUT). In these theories there are two widely spaced energy scale: the scale of elementary particle physics energy is usually about a billion electron volts, and the energy range of 10 TVO^{15} or so billion electron volts. Between these two scales

extends an extensive "energy desert", which does not show any new phenomena. However, when calculating the effects of renormalization of the two energy scales inevitably begin to stir. Loop corrections (for example, measures to quark masses) can increase the weight up to values close to the energies of TVO, which is unacceptable. "Fine-tuning" of constant interaction and mass manually can in principle solve the problem of the hierarchy, but it will require more tweaks and will look too artificial. Fortunately, Ward-Takahashi identities supersymmetric strong enough to ensure that the "theorems on nonrenormalizability" in all orders of perturbation theory. Thus, SUSY necessary for stabilization of these two masses to the scale perturbation and to prevent their mixing.

(3) Supersymmetry can shed light on the problem of the "cosmological constant." These observational astronomy indicate that contains a cosmological constant member Xyf-g, serving an amendment to the Einstein-Hilbert action is extremely small at astronomical distance scales. The problem is how to explain the almost complete disappearance of the cosmological constant without "fine

settings. " Supersymmetry is probably strong enough to ensure the vanishing of the cosmological constant in all orders of perturbation theory (as this term breaks supersymmetry). This, however, does not fully solve the problem of the cosmological constant, as we inevitably have to break supersymmetry, to achieve a range of conventional energies. (The problem is to explain the vanishing cosmological constant after the breaking of supersymmetry has already happened.)

(4) Supersymmetry removes many unwanted particles. Tachyon, which occurs in bosonic string model is eliminated, for example, measures that it violated supersymmetry. By removing these particles also reduces the divergence SUSY diagrams with higher loops. In Chapter. 5 we show that which can potentially appear divergence superstring theory associated with infrared emission and tachyon dilatons. Therefore, eliminating these particles, at the same time we eliminate possible sources of divergences.

(5) Finally, when developing a local supersymmetry gauge theory, it naturally reduces the consumption bridge quantum theory of gravity. This occurs when the rank-that the local SUSY can be determined only in the presence of gravitons (see. Appendix). Local supersymmetry thus closely connected with the general theory of relativity. Indeed, local supersymmetry successfully eliminates the divergence of the lower loop diagrams supergravity. However, the most extensive of the theories supergravitation, 0 (8) - supergravitation apparently has a divergence in the seventh loop level, which is likely to exclude supergravitation as acceptable quantum field theory. Only by combining local supersymmetry with conformal invariance of string theory, we get a large enough gauge group to eliminate,

№6 two-dimensional supersymmetry. trees

Given the difficulties associated with superparticles (such as lack of covariant quantization and algebra, closes off the mass shell), to temporarily forget about the space-time supersymmetry and discuss the two-dimensional symmetry on the world surface of the simplest of all possible actions, including free strings and free fermions. This action we can just take in the conformal gauge, but it will reveal all the essential features of a two-dimensional supersymmetry. In this formulation, with fixed-term calibration, we will impose a calibration due to the Fock space manually. Subsequently, we will present the full effect, and then we can bring these relationships, starting with locally-acting symmetric.

The calibration is performed conformal [1]

(3.2.1)

where the index a assumes values 1 and 2, serving as a label-dimensional vectors, ts-spatiotemporal index.

Note that the y / - strange object: it anticommutes Majorana spinor in two-dimensional space and a vector in the real space-time. define

$$\psi^{\mu} = \begin{pmatrix} \psi^{\mu}_{0} \\ \psi^{\mu}_{+} \end{pmatrix}; \quad \bar{\psi}^{\mu} = \psi^{\mu}\rho^{0}$$

(3.2 $$\rho^{0} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\rho^{1} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},$$

$\{P \setminus p^s\}$ = - 2 L and L .

The Lagrangian explicitly written out through the components, we obtain

- X '-X' + 1> of (Sm + da) v | /0 - 3a) Vi). (3.2.3)

2 I

Although the calibration is fixed in this action, it still retains the invariance of global transformation

8i ^ = -ip$^a d_a X$ »e. (3.2.4)

So, temporarily abandoning the attempt to construct a theory of strings with these space-time spinors, we have received a two-dimensional supersymmetric theory, which is very simple and includes free boson and anticornmuting field.

Historically, NS-R [4 - 6] theory Neveu-Schwarz Ramon was the first successful attempt to enter a dual spin model. It is also the first example of a linear supersymmetric steps [1], and soon / four-SUSY was followed by steps to point particles [7, 8].

Now that we have written our two-dimensional supersymmetric action, we trace the steps taken in the previous chapter for finding solutions of the system. Next step- find currents associated with these symmetries, then determine how these currents generate the algebra, and, finally, to put these connections in the Hilbert space. The sequence, which we adhere to in this section is a direct generalization of the steps, undertaken in the previous chapter:

Action -> Symmetry -> Currents -> Algebra -> Connections ->

-> unitary. (3.2.5)

№7 Conformal Field Theory. spin box

One of the mysteries of superstrings, to formulate its theory in two ways. The first of these model NS-R (after GSO-projection-tion) containing anticornmuting vectors, and the second model Green-Schwarz (GS) with the actual anticommutative spinors, each formulation has its own specific advantages and disadvantages. In this chapter we discuss conformal field theory, which will allow us to see the dynamic connection between the two formulations. Conformal field theory Friedan and Shenker [1] combines the best features of both theories. Conformal field theory leads to the next.

(1) Enter covariant anticornmuting spinor fields based only on the free-field. GS-formalism, in contrast, is based on a complex of interacting fields, making covariant quantization too difficult.

(2) Construct explicitly covariant wood diagrams for fermion scattering. The (NS-K) - formalizme make it virtually impossible because of the need to enter complex projection operators to eliminate the spirits. Conformal field theory replaces the clumsy projection operators free spirits Faddeev-Popov, who are easy to work with.

(3) Move from GS-wording to (NS - ^ - formulation and forth and to communicate

between them. This makes it possible to express the results obtained with the help of one of them, in terms of the other:

(4) Build covariant generators of supersymmetry. it

it is impossible to do (NS- ^ -formulation, and GS-formulation is possible only in the light cone gauge.

(5) Describe theory both sectors, NS and R, by means of the vacuum instead of using a cumbersome formalism D^{AT}* In different Hilbert spaces, based on the two vacuum[a]* '| 0>$_{NS}$ and | 0)$_R$ w_a . This is accomplished by a process called 'Vai bosonization, r. F. Construct of fermions bosons in the measuring space.

№8 fermion vertex operator

We compute the conformal weight bozonizirovannyh fields: wt (e- (1/2, ') = W9 + G) = |, (4.5.1)

Now, the missing part is found. Since bozonizirovannoe field

$_e$- *(1/2) h>* $_{R1meet\ weight}$ *s / §_* $_{we\ are\ M}$*ozhem build a real fermion vertex function of our theory:*

$$V\text{-}_m = u^a e\text{-}^{am,f} S_a e^{ik X}. \qquad (4.5.2)$$

Ova has conformal weight

$$1 + 1 + a'k^2. \qquad (4.5.3)$$

If you put the external fermion line on the mass poverhnospolozhiv

$to^2 = 0,$

We will vertex function with conformal weight 1. It specifies the required properties of interaction with the BRST-charge: up to terms that vanish on the mass shell,

$$\{6BR_{st}, K_{-1/2}\}_+ = 0. \qquad (4.5.5)$$

One of the major achievements of the conformal field theory: the construction ^ Toyaschey fermion vertex function with conformal weight 1, the YSP ° M on a tio n a n d n ° on the free fields. The key point was, ° Lzovanie wind sector to provide the missing concentration ^ RMnogo weight 3/8.

Though now, our goal has been achieved, we are still not out of the chaschts We mentioned above in the preceding paragraph, there is * ODC difficulty with the infinite sea boson. It turns out that # esl substitute this fermion vertex function in the matrix element boson scattering matrix, we get 0:

$$<\dots K._{12} \dots V\text{-}U2, \dots) = 0. \qquad (4.5.6)$$

What we need is, of course, the vertex function with brass charges + 1/2, which could be reduced with a charge -1/2 originating from fermion vertex function. This new vertex function Vl {l should anticommute with BRST-charge up to terms that vanish on the mass shell. It is easy to show that every vertex function

$$V = [QBRST, F] \qquad (4.5.7)$$

F gives an arbitrary vanishing anticommutation ratio with BRST-charge since Q nilpotent. However, all such spurion function. These states are not zero and the interaction-exist with physical states | R), which satisfy the soot-wearing

$$s_{By\ 8\ m} | A> = 0, \qquad (4.5.8)$$

so that they can not be used as a vertex functions. They just give a zero matrix elements of the physical sector of the theory. However, there is one vertex function for which this argument does not apply:

Pi / 2 = 2 [e $_B$ RST, ^. $_{12}$]. (4.5.9)

Under normal circumstances, one would expect that such a vertex function is also spurion and does not interact with the physics-cal states of the system. However, £ K1 / 2, as we pointed out earlier, is not part of the irreducible Fock space theory, and therefore we can not simply say that the switch vanishes while reducing physical condition. This vertex function optionally vanishes because

e_BRST = ^ | 0> = 0. (4.5.

Making calculations, we find that this function is equal to the vertex

Vii2 = u «(k) e ^{ikX}*le <* $^{l f2)}$** (DX*

»+ \ ik-wfHyJrtSP + (4.5.

Such is correct fermion vertex functions * 'cutting with K_1 / 2. nd yet, here we are faced with a rather alarming

problem. Now we have is too many opportunities * vertex functions!

For example, we might as well write

Ks / 2 ® CQBRST, 5 ^ 1/2].

K5 / 2 = ShvK8T, ^ 3/2] (4.5.12)

in fact, there are infinitely many such vertex functions, each associated with its own, non-equivalent others, boson-gjiM vacuum sea. Of course, such an abundance of causes confusion. However, we can show that we use quite a ¥ ^ 2 0 Y 1/2 "and with e other peaks function will not allow any new

^ Matrix element. We want to show that we have the following identity:

<... *V*- $_{l/2}$*(u* $_v$ z $_x$), ..., *Uz* $_2$*(and* $_2$*to* $_2$*,* z $_2$) ...>

V $_{1/2}$*(u* $_{l9}$ *k* $_v$

= <... z $_x$) ..., V-1/2 («2» k $_2$. z $_2$) ...>. (4.5.13)

Proof that we can simply replace arbitrarily wind indices -1/2 to +1/2 or vice versa at the vertex functions, includes a fairly subtle reasoning allow ne-a transition from a small irreducible Fock space (which does not include zero mode field £) Fok reducible to large-th space (which includes this mode) and vice versa.

№9 amplitude and space Teichmuller

One of the most attractive features of string theory is the possibility of creating a theory of gravity that would have been completely Dyanitnoy and thus independent from the usual renormalization theory. String theory is capable of such an approach, in which Derbe can be formulated as a finite quantum theory of gravity. Of particular interest is the mechanism by which achieved elimination of all potential divergences, namely the use of topological considerations for eliminating certain types of divergences. We once again convinced of the immense power of symmetry, built-in string model. We show, for example, potentially divergent diagrams are topologically equivalent to the emission effective dilaton. Therefore, eliminating dvlaton of theory, we have a theory that is free from any apparent divergences. Thus, mechanisms, We never before encountered in quantum field theory, using the odds-formalism to point particles.

So far we have only developed the first quantized theory of interacting strings without loops. In this way, of course, impossible to obtain a unitary theory. Euler beta function, as we have shown above, has poles in the z-plane on the real axis without imaginary parts or sections and, therefore, such a theory describes only tree diagrams. First attempts were made to modify the initial beta function, adding to the weight of the imaginary part of the resonances:

js - Mj + iTj

Fshako thus inevitably destroyed the remarkable properties of "C-function.

The correct way to make a model of a unitary eventually H ^ Dlozhili Kikkawa, Sakit and Virasoro [1] in 1969 .: add a loop, considering the beta function as the Born term of the perturbative ® ^ DHoda to the definition of the S-matrix. Actual multiloop the amplitude-vflbi calculated Kaku, Yau, Alessandrini Lovelace and [2-8].

from ! To understand, ka G series of perturbation theory are constructed, will start the time evolution operator C /, which transforms the initial state with ^ t = - oo in the final state with t = oo. S-matrix pre-

S * = </ | £ / (- oo, oo) | />. (5.1.2)

Since the operator is unitary time evolution, the very same S-matrix is unitary:

$S^f S = SS^f = \backslash.$ (5.1.3)

In matrix form this is written as

(5.1.4)

where different n are the complete set of intermediate states. If

select a state corresponding to the absence of scattering, we are ^ we obtain the matrix:

S = \ - iT. (5.1.5)

Then

i (T- T) = TTK (5.1.6)

If we take the matrix elements for the scattering of many-particle initial state

</ |, Converted into the final many-particle states | y), we obtain

1 7: L = - £ </ | G | | "> <1» | G | y> (5.1.

(See. Fig. 5.1). If we imagine the scattering amplitude four strings like </ | T \ j), it is clear that we need to combine different four-point function to get the next order of perturbation theory. Since strings can be twisted, sweeping the two-dimensional surface, a set of Feynman diagrams for loop greater than for simple planar graphs. In fact, as shown in Fig. 5.2, there are three types of charts that can be constructed by using the optical theorem.

№10 Field theory in the light cone gauge

Previous intuitions lead us to affirm the existence of four-stringed interaction [1], in which the two strings can interact in their interior points and change the local topology immediately. At first glance, the existence of the corresponding diagram in the method of the Neumann functions is not obvious, since the upper half is always transformed into a planar configuration. However, the presence of members four-stringed interaction in Veneziano amplitude can be seen, carefully check the domain of integration variables Koba-Nielsen.

We start with a four-stringed display and believe xx = 1, x2 = oo, Xs = 0 and jc4 = N ;. Then it takes the form of a map

$$p = a_t \ln (z - 1) + a_3 \ln z + a_4 \ln (z - x). \qquad (6.7.1)$$

DL YaTOGOchtoby find its singularity, we put

$$? = 0. \qquad (6.7.2)$$

dz

The solution of this equation gives a turning point for our conversion:

z 2
(1 - Y 0 + (b ~ B) X ± D 1/2} (6?3
\
<*
a i
1 ȧ_x
<
In
2 a_x

/ $x^2(y_2 - the_x)^2$ 2 {+ 2y, y_2 - the_x - (B -?

Generally, two solutions for the equation of the turning points of a n d d e e e I n m u e ± square root signs, show that the Riemann surface there are two turning points having URD * relative freedom of movement to the friend. The s- and / - channel diagrams world ° v1 strings surface deform smoothly into each other. However, k0 in the p-plane of the two interaction points coincide imaginary portions> interesting thing happens.

№11 Field theory BRST

The great advantage of field theory in the light-cone gauge, as we have seen, lay in the fact that it was clearly a unitary free spirits and could play the string amplitudes from a single action. There was no need to contact the intuition to build a unitary matrix.

However, this theory is flawed. The fact that it would be desirable to have a covariant description, which involved all the strings calibration. Therefore, our next step in the development of the field theory of strings is the use BRST techniques for covariant description of string fields. BRST formalism force is due to the fact that op allows you to reformulate field theory of strings in a fully covariant form with the introduction of the Faddeev-Popov ghosts.

We move on to the field theory in the same way that Feynman went to the Schrödinger equation of the classical primary-quantized theory. Starting with the formalism of BRST first quantized theory, we then obtain a description of the language of the functional field. Particular attention should be paid to the fact that the BRST field theory, like the formalism of the light cone, still remains a theory in fixed-term calibration. Since the theory BRST is derived from the first quantized fixed gauge theory, we find quite strange objects first quantized theory, inherited second quantized theory, such as the Faddeev - Popov wind number parameterization midpoints and parameterization length.

This too is a certain irony. Initially, the field theory in the light cone gauge was introduced to present a consistent and comprehensive formalism, in which you can express a complete theory. Unfortunately, attempts to make the model covariance led to the creation of two competing covariant string field theory in the BRST. These two theories about BRST- a tio n s and are in a completely different string topologies, and can not see any connection between them, except that they both can be successful in about a n d s o and - carry model Veneziano.

All these difficulties will be eliminated when we finally we reach D ° geometric variant of the field theory in the next chapter.

Earlier we saw that the Gupta-Bleuler formalism instead of resolved "* constraint equations (as in the formalism of the light cone) imposes svya directly to state:

L "| (r> = 0; n> 0.0 ll)

otveyano expect that action allows to spread to all 26 ^ 01oyaentam theory. It is, however, not the case, if we take care to consider te constraints imposed on state vectors. The result of applying 001 the previous equation is to destroy the state of wind

I spektre-

In addition, there is another way to eliminate wind-in-consisting act by analogy with the gauge theory. Instead nalozhesvyazey on Hilbert space, we require that the action of Kilo invariant under the transformations that convert a field F in the state of spirits:

5 | 0> = LL _ JA "> . (7.1.2)

P

Note that the state of L _ "| A) is the ghost states (see (2.9.3.)). This choice is motivated by the analogy with electromagnetism, where there is a gauge symmetry:

= (7.1.3)

Action for Maxwell field remains unchanged when adding Toll Ar brass field d ^ X. We now prove that the string does contain a variation of the gauge variations in the fields of Maxwell and the linearized gravitational field. As we saw in (6.3.19), the functional field | F> represents the sum of all possible states of a string. Expanding the fields on their components:

| A) = A (x) 10) + ... (7.1.4)

Li = k_t-*ai* + ...

Substituting this expansion into a gauge transformation string field (7.1.2), we find

H fl - il O> + ... = ^ 5, 1 0) + ...

Equating terms of the expansion, we obtain (7.1.3). Thus, we have deduced Maxwell field gauge transformation, [1-3] plevu expanding functionality.

Similarly, it is also possible to show that the sector is closed ^ VH comprises linearized gravitational field. We require

^{8}IV> = X b - "| A_n > + L- "| A">, (7.1.5)

and

feature is a second independent Hilbert space. * Yves different this expression, we arrive at

^{l}> -787

(7.1.6)

+ ...

k

1 -

x = + ••••

Obedinyayavse together, get

(7

Yu> + - = Yu> + (y - v) + ... l.7)

Equating the coefficients, we find

=

E ^ ku (7.1.8)

t. e. restored the original variation linearized gras-gravitational field.

№12 Geometric field theory of strings

As we have seen, the development of string theory in the last 20 years occurred in a direction directly opposite to the direction of development of general relativity. To a large extent this explains why we are now busy looking for the fundamental geometric formulation of string theory.

General relativity was discovered by Einstein, who first tried to explain the most important physical principle - the principle of equivalence, and then postulated a geometric formulation of explaining the general covariance. The next step was for the structure of a single-action, satisfying these principles. Further stages in the development of the general theory of relativity have been associated with the creation of the classical theory of Riemannian manifolds. Finally, attempts to quantize this theory have been made. Thus, the historical development of the scheme general theory of relativity-ness is as follows:

The geometry of the classical theory of action -> Quantum Theory.

Compared with this circuit string theory developed in reverse. Its development begins with the accidental discovery Veneziano-Born member constructing quantum loops, leading to classes cal Nambu-Goto, then to the action in the light-cone gauge and, finally, to the geometric O attempts steps:

Quantum theory -> Classical Theory of Action Geometry.

Only recently, with the revival of interest in string theory, trying to work together to finalize its development have been made. As we have emphasized, it is not just an academic question. Ultimately, the success or failure of string theory will be determined on the basis of

smozhetlionavybratpodhodyaschiykvantovyyvakuumsredidesyatkovtysyach opportunities. Thus, only in the framework of

present

field theory it will be possible to solve the urgent problem of string theory, "namely nonperturbative symmetry breaking, which reduces the 10-mer

vacuum to the four.

In fact, there are only two different ways in which 0 can be derived field theory of strings.

(A) First, you can try to get the effect of the field theory of "building on the first quantized steps Nambu, so

way that Feynman gave the Schrödinger equation of the classical theory of non-relativistic particles. Such a strategy "CNI have

Up "includes extension reasonable assumptions on the basis of features-SRI first quantized theory. The disadvantage of this approach lies in the fact that it always breaks the gauge invariance of the theory (t. E. We must choose the calibration light-Vågå cone or conformal gauge BRST). This means that you need to assume the existence of arbitrarily defined-tion fields (eg "wind Faddeev Popov fields"). These fields are present in the first quantized approach, with fixed settings, but their appearance in the second quantized field-ing theory is a strange and unnatural. Therefore, in such a theory, this action looks fanciful. In other words, the BRST approach by itself is not a natural principle,

(B) Secondly, in the framework of the geometric approach can bring the whole field theory, highlighting the fundamental physical principles. This method of "top-down", inheriting the spirit of Yang-Mills theory and general relativity. Now we start with the unity-term local gauge group, the introduction of which is based on simple physical principles, and require action to be invariant with respect to transformations of the group. Bases Nye problem of this method is to separate consideration gauge group string theory, finding its Mykh-irreducible representations of curvature and the action itself.

In this chapter we look at the most advanced version of the geometric theory. We will stick to the analogy with the general theory of relativity and the theory-ness

Yang-Mills, which can be derived from two pro-grained principles of geometric origin - global and locally-symmetry.

Global symmetry. Theory must describe the spread

helicity corresponding pure fields A £ 1 with a spin and spin g to about 2, transforming both irreducible representations of SU (JV) and the Lorentz group.

The local symmetry. The action to be locally invariant under the action of SU (N) and generally covariant.

№13 Anomalies and Atiyah-Singer

Videale we would like to see a truly unified field theory of all known interactions satisfy at least two criteria:

(1) It should be based on simple physical assumptions, expressed in terms of a new geometry that will allow no more than one interaction constant.

(2) It should result in a final gravity theory connected

with minimal SU (3) x SU (2) x U (1) -model particle interactions.

So far in this book we have only begun to explore the possibility of the first, indicating that the second-quantized field theory, based on the two physical principles exist. However, achieve string theory, which we have described so far have been purely formal. If we can not compare the theory with the experimental data, it is however elegant it may be, it will have to be discarded. The true test for a unified field theory is the requirement that at low energies it could reproduce

maintain the known experimental data.

The problem, however, lies in the fact that the dimensional reduction

10-dimensional theory to 4 measurements can only occur neper-turbativno. For any finite order perturbation theory, the dimension of time-space-time seems quite unchanged-tion. Generally speaking, the field theory provides the only reliable formalism in which nonperturbative calculations can be carried out as the first quantized formalism is necessarily perturbative. Unfortunately, we do not yet understand how to perform nonperturbative

calculations in string-theory mainly because h t 0 string field theory is still in its infancy. For example, the physics are not able to calculate

stability of any of the classic vacuum solutions. Therefore, we will not touch the quantum stability in Part III of this book and of a p e d h o h m ^ and exclusively on the classical equations of motion.

It's amazing that with such severe restrictions already the very first attempts to investigate the experimental investigation of the classical string-theory was given a lot of new phenomenological consequences, leading us beyond TBO. In Part III, we first brought the question whether string theory is consistent with the results of standard TBO? Especially we are interested in, whether it is repro-lime grand unification theory with the gauge groups

SU (5)> Ltd.) and L and Ev- In this respect, string theory has managed to stitch-specific

success. We will show in Chapter. 11 that, for example,

heterotic string E8 E8 x can be easily reduced class-to-classical methods theory with gauge group E6 which has solutions with chiral fermions and acceptable from the standpoint of phenomenology GUT.

But we must also demand that string theory goes beyond standard phenomenology TVO. Namely, we have to post - twist before it the following questions related to the string model:

(1) Can she explain the three generations of chiral fermions?

(2) Can she explain the experimental results on proton decay?

(3) Can she explain the smallness of the electron mass?

(4) Can she explain the vanishing cosmological constant after supersymmetry breaking?

Although it is still too early to say this categorically, yet there is evidence that string theory is meaningful and that it relies on the mathematical apparatus, with which you can get the answers to the above questions. In particular, substantially topology is used, so that the basic phenomenological concepts, such as the number of generations, is now reformulated in the language of topology.

Topology is the one new mathematical tool that will enable us to go beyond the standard phenomenology TVO.

№14 Heterotic string compactification

One of the major problems faced by the string theory, is to describe the transition from 26- and 10-dimensional theories in a realistic 4-dimensional theory. Until such a dimensional reduction will not be performed, the theory can not claim to be any serious description of physical reality.

While dimensional reduction is not carried out within the framework of the field theory, the best thing you can do - is to consider classical solutions, describing the spontaneous compactification of additional changes

rhenium. In this chapter we will investigate the string with heterotic groups E8 (x) Es and Spin (32) / Z2, arising from the compactification of 26-dimensional space to 10 measurements. As we saw in the previous chapter, the reduction of the anomalies is possible for groups 0 (32) and E8 (x) E8. We have seen, however, that the method of Chan-Paton does not work for the exceptional groups. Therefore, to obtain a pattern group must E8 E8 ®

apply a different method using compactification on self-dual lattice.

Although

theory heterotic string is a theory of closed strings, it has a field of super-Yang-Mills theory, usually occurs in the open string sector for strings of type I. In theory, heterotic string used deceptively simple identity

2 6 = 10 + 16. (10.1.1)

This means that the compactification 26-dimensional string to a 10-dimensional stays 16 additional measurements that can be placed on the torus generated root grating group E8® E8 that pri-

drives are known to free from anomalies theory. This observation was made by Freund [1].

The heterotic string uses the fact that a closed string has two separate sectors: the right and left. The law, all the functions depend on the sector a + t, and on the left by a -m. This splitting is essentially used in heterotic string theory. The word "heterosis" means "hybrid force (energy)." Here, it means that

^ Balanced approach to the left and right modes of the hybrid theory leads to considerably more complicated than previously studied superstring ^ Ipa I and II. It has been shown that this theory has no tachyon,

"^ Abnormal and is the ultimate in one-loop approximation.

№15 About the theory of supersymmetry

There are at least four ways to formulate the theory of supergravity:

(1) Exploded view. This method is widely ispolzuettehniku trial and error, but it gives the most explicit form

actions. (2) Introduction via curvature. This method is based on the theory of groups and the analogy with the Yang-Mills theory. (3) The tensor calculus. It gives the exact rules of multiplication supersymmetry representations.

(4) Superspace. This is the most elegant formulation of supergravity, but it is also the most difficult. Superspace formulation for high values of N that are not known yet, since restrictions on the torsion too difficult to resolve. We will focus on the method of curvature, since it resembles the construction of Yang-Mills theory, which we have used so por.Poskolku group Osp (l / 4) has 14 generators define 14 fields Osp band connectivity (l / 4) by the expression K = (< , <, <). (Section 5.1)

Then the global variation of the connection fields is 5 / *? = / 2 e q (5.2)

Where

= (8 °, 8o, 8a). (Section 5.3)

The covariant derivative is now given by the expression

= Al + e; pa + <1a Mob + ^ <2 a (a> b). (Section 5.4)

Under the influence of the local gauge transformation, these fields are transformed as M ^ d ^ + h ^ fb. (Clause 5.5)

Now take two switch covariant derivatives: [V,, 4 v 2 = R ^ M a, (item 5.6)

Where

K = number - dL + h * hcJtB. (Item 5.7)

The componentwise recording obtain R & (M) = ^ cof + <CDc

v

b - (jj, «- • v), (item 5.8)

Km = + Y v <Go b - (p ^ v).

It is easy to show that there is a variation of the curvature 5 <= <s 7 c B. (Item 5.9)

Action supergravity theory is now written in the form

*S = lctxz ^{R, v (M) abR, a (M) cdzabcd* + / S) aYara (e) P (U5aR} • (P.5.YU)

If we now vary the effect of this, it appears that it is not completely invariant, unless you put R »AP) a = 0. (P.5.11)

action written above is invariant up to ^ bsoGD member). (Section 5.12 ADVANCE MENU_SYSTEM)

However, since we have imposed this link (P.5.11) from the outset, this action is in fact completely invariant under this transformation.

This link seems very artificial, until one realizes that it is actually equivalent to the vanishing of the covariant derivative of the notebook (P.2.31). Therefore, we choose a notebook with a derivative of zero to get the invariant action.